

# The extension package `curve2e`\*

Claudio Beccari

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```
i*package;
1 \NeedsTeXFormat{LaTeX2e}
2 \</package>
3 \<*driver>
4 \ProvidesFile{curve2e.dtx}%
5 \</driver>
6 \<+package>\ProvidesPackage{curve2e}%
7 [2006/11/27 v.1.00 Extension package for pict2e]
8 \<*package>
i/package;
```

## Abstract

This file documents the `curve2e` extension package to the recent implementation of the `pict2e` bundle that has been described by Lamport himself in the second edition of his  $\text{\LaTeX}$  handbook.

This extension redefines a couple of commands and introduces some more drawing facilities that allow to draw circular arcs and arbitrary curves with the minimum of user intervention. This beta version is open to the contribution of other users as well as it may be incorporated in other people's packages. Please cite the original author and the chain of contributors.

## 1 Package `pict2e` and this extension `curve2e`

Package `pict2e` was announced in issue 15 of `latexnews` around December 2003; it was declared that the new package would replace the dummy one that has been

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accompanying every release of  $\text{\LaTeX}$  2 $\epsilon$  since its beginnings in 1994. The dummy package was just issuing an info message that simply announced the temporary unavailability of the real package.

Eventually G   lein and Niepraschk implemented what L  mpert himself had already documented in the second edition of his  $\text{\LaTeX}$  handbook, that is a  $\text{\LaTeX}$  package that contained the macros capable of removing all the limitations contained in the standard commands of the original `picture` environment; specifically:

1. the line and vector slopes were limited to the ratios of relatively prime one digit integers of magnitude not exceeding 6 for lines and 4 for vectors;
2. filled and unfilled full circles were limited by the necessarily bounded number of specific glyphs contained in the special  $\text{\LaTeX}$  `picture` fonts;
3. quarter circles were also limited in their radii for the same reason;
4. ovals (rectangles with rounded corners) could not be too small because of the unavailability of small radius quarter circles, nor could be too large, in the sense that after a certain radius the rounded corners remained the same and would not increase proportionally to the oval size.
5. vector arrows had only one possible shape besides matching the limited number of vector slopes;
6. for circles and inclined lines and vectors there were available just two possible thicknesses.

The package `pict2e` removes most if not all the above limitations:

1. line and vector slopes are virtually unlimited; the only remaining limitation is that the direction coefficients must be three-digit integer numbers; they need not be relatively prime;
2. filled and unfilled circles can be of any size;
3. ovals can be designed with any specified corner curvature and there is virtually no limitation to such curvatures; of course corner radii should not exceed half the lower value between the base and the height of the oval;
4. there are two shapes for the arrow tips; the triangular one traditional with  $\text{\LaTeX}$  vectors, or the arrow tip with PostScript style.
5. the `\linethickness` command changes the thickness of all lines, straight, curved, vertical, horizontal, arrow tipped, et cetera.

This specific extension adds the following features

1. commands for setting the line terminations are introduced; the user can choose between square or rounded caps; the default is set to rounded caps;
2. the `\line` macro is redefined so as to allow integer and fractional direction coefficients, but maintaining the same syntax as in the original `picture` environment;

3. a new macro `\Line` is defined so as to avoid the need to specify the horizontal projection of inclined lines;
4. a new macro `\LINE` joins two points specified with their coordinates; of course there is no need to use the `\put` command with this line specification;
5. similar macros are redefined for vectors; `\vector` redefines the original macro but with the vector slope limitation removed; `\Vector` gets specified with its two horizontal and vertical components; `\VECTOR` joins two specified points (without using the `\put` command) with the arrow pointing to the second point;
6. a new macro `\polyline` for drawing polygonal lines is defined that accepts from two vertices up to an arbitrary (reasonably limited) number of them;
7. a new macro `\Arc` is defined in order to draw an arc with arbitrary radius and arbitrary angle amplitude; this amplitude is specified in sexagesimal degrees, not in radians;
8. two new macros are defined in order to draw circular arcs with one arrow at one or both ends;
9. a new macro `\Curve` is defined so as to draw arbitrary curved lines by means of third order Bézier splines; the `\Curve` macro requires only the curve nodes and the direction of the tangents at each node.

In order to make the necessary calculations many macros have been defined so as to use complex number to manipulate point coordinates, directions, rotations and the like. The trigonometric functions have also been defined in a way that the author believes to be more efficient than that implied by the `trig` package; in any case the macro names are sufficiently different to accommodate both definitions in the same  $\text{\LaTeX}$  run.

Many aspects of this extension could be fine tuned for better performance; many new commands could be defined in order to further extend this extension. If the new service macros are accepted by other  $\text{\TeX}$  and  $\text{\LaTeX}$  programmers, this beta version could become the start for a real extension of the `pict2e` package or even become a part of it.

For this reason I suppose that every enhancement should be submitted to Gäßlein and Niepraschk who are the prime maintainers of `pict2e`; they only can decide whether or not to incorporate new macros in their package.

## 2 Acknowledgements

I wish to express my deepest thanks to Michel Goossens who spotted some errors and very kindly submitted them to me so that I was able to correct them.

## 3 Source code

### 3.1 Some preliminary extensions to the `pict2e` package

The necessary preliminary code has already been introduced. Here we require the `color` package and the `pict2e` one; for the latter one we make sure that a

sufficiently recent version is used.

```
9 \RequirePackage{color}
10 \RequirePackageWithOptions{pict2e}[2004/06/01]
```

Next we define the line terminators and joins; the following definitions work correctly if the `dvips` or the `pdftex` driver are specified; probably other modes should be added so as to be consistent with `pict2e`.

```
11 \ifcase\pIIe@mode\relax
12 \or %Postscript
13 \def\roundcap{\special{ps:: 1 setlinecap}}%
14 \def\squarecap{\special{ps:: 0 setlinecap}}%
15 \def\roundjoin{\special{ps:: 1 setlinejoin}}%
16 \def\beveljoin{\special{ps:: 2 setlinejoin}}%
17 \or %pdf
18 \def\roundcap{\pdfliteral{1 J}}%
19 \def\squarecap{\pdfliteral{0 J}}%
20 \def\roundjoin{\pdfliteral{1 j}}%
21 \def\beveljoin{\pdfliteral{2 j}}%
22 \fi
```

The next macros are just for debugging. With the `tracing` package it would probably be better to define other macros, but this is not for the users, but for the developers.

```
23 \def\TRON{\tracingcommands\tw@ \tracingmacros\tw@}%
24 \def\TROF{\tracingcommands\z@ \tracingmacros\z@}%
```

Next we define some new dimension registers that will be used by the subsequent macros; should they be already defined, there will not be any redefinition; nevertheless the macros should be sufficiently protected so as to avoid overwriting register values loaded by other macro packages.

```
25 \ifx\undefined\@tdA \newdimen\@tdA \fi
26 \ifx\undefined\@tdB \newdimen\@tdB \fi
27 \ifx\undefined\@tdC \newdimen\@tdC \fi
28 \ifx\undefined\@tdD \newdimen\@tdD \fi
29 \ifx\undefined\@tdE \newdimen\@tdE \fi
30 \ifx\undefined\@tdF \newdimen\@tdF \fi
31 \ifx\undefined\defaultlinewidth \newdimen\defaultlinewidth \fi
```

It is better to define a macro for setting a different value for the line and curve thicknesses; the `\defaultlinewidth` should contain the equivalent of `\@wholewidth`, that is the thickness of thick lines; thin lines are half as thick; so when the default line thickness is specified to, say, 1pt, thick lines will be 1pt thick and thin lines will be 0.5pt thick. The default whole width of thick lines is 0,8pt, but this is specified in the kernel of `LATEX` and/or in `pict2e`.

```
32 \newcommand\defaultlinethickness[1]{\defaultlinewidth=#1\relax
33 \def\thicklines{\linethickness{\defaultlinewidth}}%
34 \def\thinlines{\linethickness{.5\defaultlinewidth}}%
35 \thinlines\ignorespaces}
```

The `\ignorespaces` at the end of this and the subsequent macros is for avoiding spurious spaces to get into the picture that is being drawn, because these spaces introduce picture deformities often difficult to spot and eliminate.

### 3.1.1 Improved line and vector macros

The new macro `\Line` allows to draw an arbitrary inclination line as if it was a polygon with just two vertices. This line should be set by means of a `\put` command so that its starting point is always at a relative 0,0 coordinate point. The two arguments define the horizontal and the vertical component respectively.

```
36 \def\Line(#1,#2){\pIIE@moveto\z@\z@
37   \pIIE@lineto{#1\unitlength}{#2\unitlength}\pIIE@strokeGraph}%
```

A similar macro `\LINE` operates between two explicit points with absolute coordinates, instead of relative to the position specified by a `\put` command; it resorts to the `\polyline` macro that is to be defined in a while.

```
38 \def\LINE(#1)(#2){\polyline(#1)(#2)}%
```

The `\line` macro is redefined by making use of a new division routine that receives in input two dimensions and yields on output their fractional ratio. The beginning of the macro definition is the same as that of `pict2e`:

```
39 \def\line(#1)#2{\begingroup
40   \@linelen #2\unitlength
41   \ifdim\@linelen<\z@\@badlinearg\else
```

but as soon as it is verified that the line length is not negative, things change remarkably; in facts the machinery for complex numbers is invoked: `\DirOfVect` takes the only macro argument (that actually contains a comma separated pair of fractional numbers) and copies it to `\Dir@line` (an arbitrarily named control sequence) after renormalizing to unit magnitude; this is passed to `GetCoord` that separates the two components into the control sequences `\d@mX` and `\d@mY`; these in turn are the values that are actually operated upon by the subsequent commands.

```
42   \expandafter\DirOfVect#1to\Dir@line
43   \GetCoord(\Dir@line)\d@mX\d@mY
```

The normalized vector direction is actually formed with the directing cosines of the line direction; since the line length is actually the horizontal component for non vertical lines, it is necessary to compute the actual line length for non vertical lines by dividing the given length by the magnitude of horizontal cosine `\d@mX`, and the line length is accordingly scaled:

```
44   \ifdim\d@mX\p@=\z@\else
45     \DividE\ifdim\d@mX\p@<\z@-\fi\p@ by\d@mX\p@ to\sc@lelen
46     \@linelen=\sc@lelen\@linelen
47   \fi
```

Finally the `moveto`, `lineto` and `stroke` language keywords are invoked by means of the internal `pict2e` commands in order to draw the line. Notice that even vertical lines are drawn with the “PostScript” commands instead of resorting to the dvi low level language that was used both in `pict2e` and in the original `picture` commands; it had a meaning in the old times, but it certainly does not have any when lines are drawn by the driver that drives the output to a visible document form, not by  $\text{\TeX}$  the program.

```
48   \pIIE@moveto\z@\z@
49   \pIIE@lineto{\d@mX\@linelen}{\d@mY\@linelen}%
50   \pIIE@strokeGraph
51 \fi
52 \endgroup\ignorespaces}%
```

The new macro `\GetCoord` splits a vector (or complex number) specification into its components:

```
53 \def\GetCoord(#1)#2#3{%
54 \expandafter\SplitNod@\expandafter(#1)#2#3\ignorespaces}
```

But the macro that does the real work is `\SplitNod@`:

```
55 \def\SplitNod@(#1,#2)#3#4{\edef#3{#1}\edef#4{#2}}%
```

The redefinitions and the new definitions for vectors are a little more complicated than with segments, because each vector is drawn as a filled contour; the original `pict2e` macro checks if the slopes are corresponding to the limitations specified by Lamport (integer three digit signed numbers) and sets up a transformation in order to make it possible to draw each vector as an horizontal left-to-right arrow and then to rotate it by its angle about its tail point; actually there are two macros for tracing the contours that are eventually filled by the principal macro; each contour macro draws the vector with a  $\text{\LaTeX}$  or a PostScript arrow whose parameters are specified by default or may be taken from the parameters taken from the `PSTricks` package if this one is loaded before `pict2e`; in any case we did not change the contour drawing macros because if they are modified the same modification is passed on to the arrows drawn with the `curve2e` package redefinitions.

Because of these features the redefinitions and the new macros are different from those used for straight lines.

We start with the redefinition of `\vector` and we use the machinery for vectors (as complex numbers) we used for `\line`.

```
56 \def\vector(#1)#2{%
57 \begingroup
58 \GetCoord(#1)\d@mX\d@mY
59 \@linelen#2\unitlength
```

As in `pict2e` we avoid tracing vectors if the slope parameters are both zero.

```
60 \ifdim\d@mX\p@=\z@\ifdim\d@mY\p@=\z@\@badlinearg\fi\fi
```

But we check only for the positive nature of the  $l_x$  component; if it is negative, we simply change sign instead of blocking the typesetting process. This is useful also for macros `\Vector` and `\VECTOR` to be defined in a while.

```
61 \ifdim\@linelen<\z@\@linelen=-\@linelen\fi
```

We now make a vector with the slope direction even if one or the other is zero and we determine its direction; the real and imaginary parts of the direction vector are also the values we need for the subsequent rotation.

```
62 \MakeVectorFrom\d@mX\d@mY to\@Vect
63 \DirOfVect\@Vect to\Dir@Vect
```

In order to be compatible with the original `pict2e` I need to transform the components of the vector direction in lengths with the specific names `\@xdim` and `\@ydim`

```
64 \YpartOfVect\Dir@Vect to\@ynum \@ydim=\@ynum\p@
65 \XpartOfVect\Dir@Vect to\@xnum \@xdim=\@xnum\p@
```

If the vector is really sloping we need to scale the  $l_x$  component in order to get the vector total length; we have to divide by the cosine of the vector inclination which is the real part of the vector direction. I use my division macro; since it yields a

“factor” I directly use it to scale the length of the vector. I finally memorize the true vector length in the internal dimension `@tdB`

```

66      \ifdim@d@mX\p@=\z@
67      \else\ifdim@d@mY\p@=\z@
68      \else
69      \Divide\ifdim\@xnum\p@<\z@-\fi\p@ by\@xnum\p@ to\sc@lelen
70      \@lilen=\sc@lelen\@lilen
71      \fi
72      \fi
73      \@tdB=\@lilen

```

The remaining code is definitely similar to that of `pict2e`; the real difference consists in the fact that the arrow is designed by itself without the stem; but it is placed at the vector end; therefore the first statement is just the transformation matrix used by the output driver to rotate the arrow tip and to displace it the right amount. But in order to draw only the arrow tip I have to set the `\@lilen` length to zero.

```

74 \pIIE@concat\@xdim\@ydim{-\@ydim}\@xdim{\@xnum\@lilen}{\@ynum\@lilen}%
75      \@lilen\z@
76      \pIIE@vector
77      \pIIE@fillGraph

```

Now we can restore the stem length that must be shortened by the dimension of the arrow; examining the documentation of `pict2e` we discover that we have to shorten it by an approximate amount of  $AL$  (with the notations of `pict2e`, figs 10 and 11); the arrow tip parameters are stored in certain variables with which we can determine the amount of the stem shortening; if the stem was too short and the new length is negative, we refrain from designing such stem.

```

78      \@lilen=\@tdB
79      \@tdA=\pIIE@FAW\@wholewidth
80      \@tdA=\pIIE@FAL\@tdA
81      \advance\@lilen-\@tdA
82      \ifdim\@lilen>\z@
83      \pIIE@moveto\z@\z@
84      \pIIE@lineto{\@xnum\@lilen}{\@ynum\@lilen}%
85      \pIIE@strokeGraph\fi
86      \endgroup

```

Now we define the macro that does not require the specification of the length or the  $l_x$  length component; the way the new `\vector` macro works does not actually require this specification, because  $\TeX$  can compute the vector length, provided the two direction components are exactly the horizontal and vertical vector components. If the horizontal component is zero, the actual length must be specified as the vertical component.

```

87 \def\Vector(#1,#2){%
88 \ifdim#1\p@=\z@\vector(#1,#2){#2}
89 \else
90 \vector(#1,#2){#1}\fi}

```

On the opposite the next macro specifies a vector by means of the coordinates of its end points; the first point is where the vector starts, and the second point is the arrow tip side. We need the difference as these two coordinates, because it represents the actual vector.

```

91 \def\VECTOR(#1)(#2){\begingroup

```

```

92 \SubVect#1from#2to\@tempa
93 \expandafter\put\expandafter(#1){\expandafter\Vector\expandafter(\@tempa)}%
94 \endgroup\ignorespaces}

```

The `pict2e` documentation says that if the vector length is zero the macro designs only the arrow tip; this may work with macro `\vector`, certainly not with `\Vector` and `\VECTOR`. This might be useful for adding an arrow tip to a circular arc

### 3.1.2 Polygonal lines

We now define the polygonal line macro; its syntax is very simple

`\polygonal(P0)(P1)P2)... (Pn)`

In order to write a recursive macro we need aliases for the parentheses; actually we need only the left parenthesis, but some editors complain about unmatched delimiters, so we define an alias also for the right parenthesis.

```

95 \let\lp@r( \let\rp@r)

```

The first call to `\polyline` examines the first point coordinates and moves the drawing position to this point; afterwards it looks for the second point coordinates; they start with a left parenthesis; if this is found the coordinates should be there, but if the left parenthesis is missing (possibly preceeded by spaces that are ignored by the `\ifnextchar` macro) then a warning message is output together with the line number where the missing parenthesis causes the warning: beware, this line number might point to several lines further on along the source file!

```

96 \def\polyline(#1){\beveljoin\GetCoord(#1)\d@mX\d@mY
97   \pIIE@moveto{\d@mX\unitlength}{\d@mY\unitlength}%
98   \ifnextchar\lp@r{\p@lyline}{%
99   \PackageWarning{curve2e}%
100   {Polygonal lines require at least two vertices!\MessageBreak
101   Control your polygonal line specification!\MessageBreak}%
102   \ignorespaces}}

```

But if there is a second or further point coordinate the recursive macro `\p@lyline` is called; it works on the next point and checks for a further point; if such a point exists it calls itself, otherwise it terminates the polygonal line by stroking it.

```

103 \def\p@lyline(#1){\GetCoord(#1)\d@mX\d@mY
104   \pIIE@lineto{\d@mX\unitlength}{\d@mY\unitlength}%
105   \ifnextchar\lp@r{\p@lyline}{\pIIE@strokeGraph\ignorespaces}}

```

### 3.1.3 The red service grid

The next command is very useful for debugging while editing one's drawings; it draws a red grid with square meshes that are ten drawing units apart; there is no graduation along the grid, since it is supposed to be a debugging aid and the user should know what he/she is doing; nevertheless it is advisable to displace the grid by means of a `\put` command so that its grid lines coincide with the graph coordinates multiples of 10. Missing to do so the readings become cumbersome. The `\RoundUp` macros provide to increase the grid dimensions to integer multiples of ten.

```

106 \def\GraphGrid(#1,#2){\begingroup\textcolor{red}{\linethickness{.1\p@}%
107 \RoundUp#1modulo10to\@GridWd \RoundUp#2modulo10to\@GridHt

```



```

108 \@tempcnta=\@GridWd \divide\@tempcnta10\relax \advance\@tempcnta\@ne
109 \multiput(0,0)(10,0){\@tempcnta}{\line(0,1){\@GridHt}}}%
110 \@tempcnta=\@GridHt \divide\@tempcnta10\advance\@tempcnta\@ne
111 \multiput(0,0)(0,10){\@tempcnta}{\line(1,0){\@GridWd}}\thinlines}%
112 \endgroup\ignorespaces}

```

Rounding up is useful because also the grid margins fall on coordinates multiples of 10. It resorts to the `\Integer` macro that will be described in a while.

```

113 \def\RoundUp#1modulo#2to#3{\expandafter\@tempcnta\Integer#1.??%
114 \count254\@tempcnta\divide\count254by#2\relax
115 \multiply\count254by#2\relax
116 \count252\@tempcnta\advance\count252-\count254
117 \ifnum\count252>0\advance\count252-\count252\relax
118 \advance\@tempcnta-\count252\fi\edef#3{\number\@tempcnta}\ignorespaces}%

```

The `\Integer` macro takes a possibly fractional number whose decimal separator, if present, *must* be the decimal point and uses the point as an argument delimiter. If one has the doubt that the number being passed to `\Integer` might be an integer, he/she should call the macro with a further point; if the argument is truly integer this point works as the delimiter of the integer part; if the argument being passed is fractional this extra point gets discarded as well as the fractional part of the number.

```

119 \def\Integer#1.#2??{#1}%

```

### 3.2 The new division macro

Now comes one of the most important macros in the whole package: the division macro; it takes two lengths as input values and computes their fractional ratio. It must take care of the signs, so that it examines the operand signs and determines the result sign separately conserving this computed sign in the macro `\segno`; this done, we are sure that both operands are or are made positive; should the numerator be zero it directly issues the zero quotient; should the denominator be zero it outputs a signed “infinity”, that is the maximum allowable length measured in points that  $\text{\TeX}$  can deal with. Since the result is assigned a value, the calling statement must pass as the third argument either a control sequence or an active character. Of course the first operand is the dividend, the second the divisor and the third the quotient.

```

120 \ifx\DivideE\undefined
121   \def\DivideE#1by#2to#3{%
122     \begingroup
123     \dimendef\Numer=254\relax \dimendef\Denom=252\relax
124     \countdef\Num 254\relax
125     \countdef\Den 252\relax
126     \countdef\I=250\relax
127     \Numer #1\relax \Denom #2\relax
128     \ifdim\Denom<\z@ \Denom -\Denom \Numer -\Numer\fi
129     \def\segno{}\ifdim\Numer<\z@ \def\segno{-}\Numer -\Numer\fi
130     \ifdim\Denom=\z@
131       \ifdim\Numer>\z@\def\Q{16383.99999}\else\def\Q{-16383.99999}\fi
132     \else
133       \Num=\Numer \Den=\Denom \divide\Num\Den
134       \edef\Q{\number\Num.}%
135       \advance\Numer -\Q\Denom \I=6\relax

```

```

136      \@whilenum \I>\z@ \do{\DividEDec\advance\I\m@ne}%
137      \fi
138      \xdef#3{\segno\Q}\endgroup
139  }%

```

The `\DividEDec` macro takes the remainder of the previous division, multiplies it by 10, computes a one digit quotient that postfixes to the previous overall quotient, and computes the next remainder; all operations are done on integer registers to whom the dimensional operands are assigned so that the mentioned registers acquire the measures of the dimensions in scaled points; T<sub>E</sub>X is called to perform integer arithmetics, but the long division takes care of the decimal separator and of the suitable number of fractional digits.

```

140 \def\DividEDec{\Numer=10\Numer \Num=\Numer \divide\Num\Den
141      \edef\q{\number\Num}\edef\Q{Q\q}\advance\Numer -\q\Denom}%
142 \fi

```

In the above code the `\begingroup...\endgroup` maintain all registers local so that only the result must be globally defined. The `\ifx...\fi` construct assures the division machinery is not redefined; I use it in so many packages that its better not to mix up things even with slightly different definitions.

The next two macros are one of the myriad variants of the dirty trick used by Knuth for separating a measure from its units that *must* be points, “pt”; One has to call `\Numero` with a control sequence and a dimension; the dimension value in points is assinged to the control sequence.

```

143 \ifx\undefined\@Numero%
144     {\let\cc\catcode \cc'p=12\cc't=12\gdef\@Numero#1pt{#1}}%
145 \fi
146 \ifx\undefined\Numero
147     \def\Numero#1#2{\dimen254
148 #2\edef#1{\expandafter\@Numero\the\dimen254}\ignorespaces}%
149 \fi

```

For both macros the `\ifx...\fi` constructs avoids messing up the definitions I have in several packages.

### 3.3 Trigonometric functions

We now start with trigonometric functions. We define the macros `\SinOf`, `\CosOf` and `\TanOf` (we might define also `\CotOf`, but the cotangent does not appear so essential) by means of the parametric formulas that require the knowledge of the tangento of the half angle. We want to specify the angeles in sexagesimal degrees, not in radians, so we can make accurate reductions to the main quadrants. we use the fromulas

$$\begin{aligned}
 \sin \theta &= \frac{2}{\cot x + \tan x} \\
 \cos \theta &= \frac{\cot x - \tan x}{\cot x + \tan x} \\
 \tan \theta &= \frac{2}{\cot x - \tan x}
 \end{aligned}$$

where

$$x = \theta / 114.591559$$

is the half angle in degrees converted to radians.

We use this slightly modified set of parametric formulas because the cotangent of  $x$  is a by product of the computation of the tangent of  $x$ ; in this way we avoid computing the squares of numbers that might lead to overflows. For the same reason we avoid computing the value of the trigonometric functions in proximity of the value zero (and the other values that might involve high tangent or cotangent values) and in that case we prefer to approximate the small angle function value with its first or second order truncation of the McLaurin series; in facts for angles whose magnitude is smaller than  $1^\circ$  the magnitude of the independent variable  $y = 2x$  (the angle in degrees converted to radians) is so small (less than 0.017) that the sine and tangent can be freely approximated with  $y$  itself (the error being smaller than approximately  $10^{-6}$ ), while the cosine can be freely approximated with the formula  $1 - 0.5y^2$  (the error being smaller than about  $4 \cdot 10^{-9}$ ).

We keep using grouping so that internal variables are local to these groups and do not mess up other things.

The first macro is the service routine that computes the tangent and the cotangent of the half angle in radians; since we have to use always the reciprocal if this value, we call it `\X` but ins spite of the similarity it is the reciprocal of  $x$ . Notice that parameter `#1` must be a length.

```
150 \def\g@tTanCotanFrom#1to#2and#3{%
151 \DivideE 114.591559\p@ by#1to\X \@tdB=\X\p@
```

Computations are done with the help of counter `\I`, of the length `\@tdB`, and the auxiliary control sequences `\Tan` and `\Cot` whose meaning is transparent. The iterative process controlled by `\@whilenum` implements the (truncated) continued fraction expansion of the tangent function

$$\tan x = \frac{1}{\frac{1}{x} - \frac{3}{\frac{1}{x} - \frac{5}{\frac{1}{x} - \frac{7}{\frac{1}{x} - \frac{9}{\frac{1}{x} - \frac{11}{x} - \dots}}}}}$$

```
152 \countdef\I=254\def\tan{0}\I=11\relax
153 \@whilenum\I>\z@do{%
154   \@tdC=\Tan\p@ \@tdD=\I\@tdB
155   \advance\@tdD-\@tdC \DivideE\p@ by\@tdD to\tan
156   \advance\I-2\relax}%
157 \def#2{\Tan}\DivideE\p@ by\tan\p@ to\Cot \def#3{\Cot}%
158 \ignorespaces}%
```

Now that we have the macro for computing the tangent and cotangent of the half angle, we can compute the real trigonometric functions we are interested in. The sine value is computed after reducing the sine argument to the interval  $0^\circ < \theta < 180^\circ$ ; actually special values such as  $0^\circ, 90^\circ, 180^\circ$ , et cetera, are taken care separately, so that CPU time is saved for these special cases. The sine sign is taken care separately according to the quadrant of the sine argument.

```
159 \def\SinOf#1to#2{\begingroup%
160 \@tdA=#1\p@%
161 \ifdim\@tdA>\z@%
```

```

162 \@whiledim\@tdA>180\p@\do{\advance\@tdA -360\p@}%
163 \else%
164 \@whiledim\@tdA<-180\p@\do{\advance\@tdA 360\p@}%
165 \fi \ifdim\@tdA=\z@
166 \gdef#2{0}%
167 \else
168 \ifdim\@tdA>\z@
169 \def\Segno{+}%
170 \else
171 \def\Segno{-}%
172 \@tdA=-\@tdA
173 \fi
174 \ifdim\@tdA>90\p@
175 \@tdA=-\@tdA \advance\@tdA 180\p@
176 \fi
177 \ifdim\@tdA=90\p@
178 \xdef#2{\Segno1}%
179 \else
180 \ifdim\@tdA=180\p@
181 \gdef#2{0}%
182 \else
183 \ifdim\@tdA<\p@
184 \@tdA=\Segno0.0174533\@tdA
185 \Divide\@tdA by\p@ to#2%
186 \else
187 \g@tTanCotanFrom\@tdA to\T and\Tp
188 \@tdA=\T\p@ \advance\@tdA \Tp\p@
189 \Divide \Segno2\p@ by\@tdA to#2%
190 \fi
191 \fi
192 \fi
193 \fi
194 \endgroup\ignorespaces}%

```

For the computation of the cosine we behave in a similar way.

```

195 \def\CosOf#1to#2{\begingroup%
196 \@tdA=#1\p@%
197 \ifdim\@tdA>\z@%
198 \@whiledim\@tdA>360\p@\do{\advance\@tdA -360\p@}%
199 \else%
200 \@whiledim\@tdA<\z@\do{\advance\@tdA 360\p@}%
201 \fi
202 %
203 \ifdim\@tdA>180\p@
204 \@tdA=-\@tdA \advance\@tdA 360\p@
205 \fi
206 %
207 \ifdim\@tdA<90\p@
208 \def\Segno{+}%
209 \else
210 \def\Segno{-}%
211 \@tdA=-\@tdA \advance\@tdA 180\p@
212 \fi
213 \ifdim\@tdA=\z@
214 \gdef#2{\Segno1}%

```

```

215 \else
216   \ifdim\@tdA<\p@
217     \@tdA=0.0174533\@tdA \Numero\@tempA\@tdA
218     \@tdA=\@tempA\@tdA \@tdA=-.5\@tdA
219     \advance\@tdA \p@
220     \DividE\@tdA by\p@ to#2%
221   \else
222     \ifdim\@tdA=90\p@
223       \gdef#2{0}%
224     \else
225       \g@tTanCotanFrom\@tdA to\T and\Tp
226       \@tdA=\Tp\p@ \advance\@tdA-\T\p@
227       \@tdB=\Tp\p@ \advance\@tdB\T\p@
228       \DividE\Segno\@tdA by\@tdB to#2%
229     \fi
230   \fi
231 \fi
232 \endgroup\ignorespaces}%

```

For the tangent computation we behave in a similar way, except that we consider the fundamental interval as  $0^\circ < \theta < 90^\circ$ ; for the odd multiples of  $90^\circ$  we assign the result a TeX infinity value, that is the maximum number in points a dimension can be.

```

233 \def\tanof#1to#2{\begingroup%
234 \@tdA=#1\p@%
235 \ifdim\@tdA>90\p@%
236   \@whiledim\@tdA>90\p@\do{\advance\@tdA -180\p@}%
237 \else%
238   \@whiledim\@tdA<-90\p@\do{\advance\@tdA 180\p@}%
239 \fi%
240 \ifdim\@tdA=\z@%
241   \gdef#2{0}%
242 \else
243   \ifdim\@tdA>\z@
244     \def\Segno{+}%
245   \else
246     \def\Segno{-}%
247     \@tdA=-\@tdA
248   \fi
249   \ifdim\@tdA=90\p@
250     \xdef#2{\Segno16383.99999}%
251   \else
252     \ifdim\@tdA<\p@
253       \@tdA=\Segno0.0174533\@tdA
254       \DividE\@tdA by\p@ to#2%
255     \else
256       \g@tTanCotanFrom\@tdA to\T and\Tp
257       \@tdA\Tp\p@ \advance\@tdA -\T\p@
258       \DividE\Segno2\p@ by\@tdA to#2%
259     \fi
260   \fi
261 \fi
262 \endgroup\ignorespaces}%

```

### 3.4 Arcs and curves preliminary information

We would like to define now a macro for drawing circular arcs of any radius and any angular aperture; the macro should require the arc center, the arc starting point and the angular aperture. The command should have the following syntax:

`\Arc(center)(starting point)(angle)`

If the *angle* is positive the arc runs counterclockwise from the starting point; clockwise if it's negative.

It's necessary to determine the end point and the control points of the Bézier spline(s) that make up the circular arc.

The end point is obtained from the rotation of the starting point around the center; but the `\pict2e` command `\pIle@rotate` is such that the pivoting point appears to be non relocatable. It is therefore necessary to resort to low level `TeX` commands and the defined trigonometric functions and a set of macros that operate on complex numbers used as vector scale-rotate operators.

### 3.5 Complex number macros

We need therefore macros for summing, subtracting, multiplying, dividing complex numbers, for determining their directions (unit vectors); a unit vector is the complex number divided by its magnitude so that the result is the cartesian form of the Euler's equation

$$e^{j\phi} = \cos \phi + j \sin \phi$$

The magnitude of a vector is determined by taking a clever square root of a function of the real and the imaginary parts; see further on.

It's better to represent each complex number with one control sequence; this implies frequent assembling and disassembling the pair of real numbers that make up a complex number. These real components are assembled into the defining control sequence as a couple of coordinates, i.e. two comma separated integer or fractional signed decimal numbers.

For assembling two real numbers into a complex number we use the following elementary macro:

```
263 \def\MakeVectorFrom#1#2to#3{\edef#3{#1,#2}\ignorespaces}%
```

Another elementary macro copies a complex number into another one:

```
264 \def\CopyVect#1to#2{\edef#2{#1}\ignorespaces}%
```

The magnitude is determined with the macro `\ModOfVect` with delimited arguments; as usual it is assumed that the results are retrieved by means of control sequences, not used directly.

The magnitude  $M$  is determined by taking the moduli of the real and imaginary parts, changing their signs if necessary; the larger component is then taken as the reference one so that, if  $a$  is larger than  $b$ , the square root of the sum of their squares is computed as such:

$$M = \sqrt{a^2 + b^2} = a\sqrt{1 + (b/a)^2}$$

In this way the radicand never exceeds 2 and it is quite easy taking its square root by means of the Newton iterative process; due to the quadratic convergence,

five iterations are more than sufficient. When one of the components is zero, the Newton iterative process is skipped. The overall macro is the following:

```

265 \def\ModOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
266 \@tempdima=\t@X\p@ \ifdim\@tempdima<\z@ \@tempdima=-\@tempdima\fi
267 \@tempdimb=\t@Y\p@ \ifdim\@tempdimb<\z@ \@tempdimb=-\@tempdimb\fi
268 \ifdim\@tempdima>\@tempdimb
269   \Divide\@tempdimb by\@tempdima to\@T
270   \@tempdimc=\@tempdima
271 \else
272   \Divide\@tempdima by\@tempdimb to\@T
273   \@tempdimc=\@tempdimb
274 \fi
275 \ifdim\@T\p@>\z@
276   \@tempdima=\@T\p@ \@tempdima=\@T\@tempdima
277   \advance\@tempdima\p@ %
278   \@tempdimb=\p@%
279   \@tempcnta=5\relax
280   \@whilenum\@tempcnta>\z@\do{\Divide\@tempdima by\@tempdimb to\@T
281   \advance\@tempdimb \@T\p@ \@tempdimb=.5\@tempdimb
282   \advance\@tempcnta\m@ne}%
283   \@tempdimc=\@T\@tempdimc
284 \fi
285 \Numero#2\@tempdimc
286 \ignorespaces}%

```

As a byproduct of the computation the control sequence `\@tempdimc` contains the vector or complex number magnitude multiplied by the length of one point.

Since the macro for determining the magnitude of a vector is available, we can now normalize the vector to its magnitude, therefore getting the cartesian form of the direction vector. If by any chance the direction of the null vector is requested, the output is again the null vector, without normalization.

```

287 \def\DirOfVect#1to#2{\GetCoord(#1)\t@X\t@Y
288 \ModOfVect#1to\@tempa
289 \ifdim\@tempdimc=\z@\else
290   \Divide\t@X\p@ by\@tempdimc to\t@X
291   \Divide\t@Y\p@ by\@tempdimc to\t@Y
292   \MakeVectorFrom\t@X\t@Y to#2\relax
293 \fi\ignorespaces}%

```

A cumulative macro uses the above ones for determining with one call both the magnitude and the direction of a complex number. The first argument is the input complex number, the second its magnitude, and the third is again a complex number normalized to unit magnitude (unless the input was the null complex number); remember always that output quantities must be specified with control sequences to be used at a later time.

```

294 \def\ModAndDirOfVect#1to#2and#3{%
295 \GetCoord(#1)\t@X\t@Y
296 \ModOfVect#1to#2%
297 \Divide\t@X\p@ by\@tempdimc to\t@X \Divide\t@Y\p@ by\@tempdimc to\t@Y
298 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%

```

The next macro computes the magnitude and the direction of the difference of two complex numbers; the first input argument is the minuend, the second is the subtraend; the output quantities are the third argument containing the magnitude

of the difference and the fourth is the direction of the difference. The service macro `\SubVect` executes the difference of two complex numbers and is described further on.

```
299 \def\DistanceAndDirOfVect#1minus#2to#3and#4{%
300 \SubVect#2from#1to\@tempa \ModAndDirOfVect\@tempa to#3and#4\relax
301 \ignorespaces}%
```

We now have two macros intended to fetch just the real or, respectively, the imaginary part of the input complex number.

```
302 \def\XpartOfVect#1to#2{%
303 \GetCoord(#1)#2\@tempa
304 \ignorespaces}%
305 %
306 \def\YpartOfVect#1to#2{%
307 \GetCoord(#1)\@tempa#2\relax
308 \ignorespaces}%
```

With the next macro we create a direction vector (second argument) from a given angle (first argument).

```
309 \def\DirFromAngle#1to#2{\CosOf#1to\t@X%
310 \SinOf#1to\t@Y\MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

Sometimes it is necessary to scale a vector by an arbitrary real factor; this implies scaling both the real and imaginary part of the input given vector.

```
311 \def\ScaleVect#1by#2to#3{\GetCoord(#1)\t@X\t@Y
312 \@tempdima=\t@X\p@ \@tempdima=#2\@tempdima\Numero\t@X\@tempdima
313 \@tempdima=\t@Y\p@ \@tempdima=#2\@tempdima\Numero\t@Y\@tempdima
314 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

Again, sometimes it is necessary to reverse the direction of rotation; this implies changing the sign of the imaginary part of a given complex number; this operation produces the complex conjugate of the given number.

```
315 \def\ConjVect#1to#2{\GetCoord(#1)\t@X\t@Y
316 \@tempdima=-\t@Y\p@\Numero\t@Y\@tempdima
317 \MakeVectorFrom\t@X\t@Y to#2\ignorespaces}%
```

With all the low level elementary operations we can now proceed to the definitions of the binary operations on complex numbers. We start with the addition:

```
318 \def\AddVect#1and#2to#3{\GetCoord(#1)\tu@X\tu@Y
319 \GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@
320 \advance\@tempdima\td@X\p@ \Numero\t@X\@tempdima \@tempdima\tu@Y\p@
321 \advance\@tempdima\td@Y\p@ \Numero\t@Y\@tempdima
322 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

Then the subtraction:

```
323 \def\SubVect#1from#2to#3{\GetCoord(#1)\tu@X\tu@Y
324 \GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@
325 \advance\@tempdima-\td@X\p@ \Numero\t@X\@tempdima \@tempdima\tu@Y\p@
326 \advance\@tempdima-\td@Y\p@ \Numero\t@Y\@tempdima
327 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
```

For the multiplication we need to split the operation according to the fact that we want to multiply by the second operand or by the complex conjugate of the second operand; it would be nice if we could use the usual postfix asterisk notation for the complex conjugate, but I could not find a simple means for doing so; therefore I use the prefixed notation, that is I put the asterisk before the



second operand. The first part of the multiplication macro just takes care of the multiplicand and then checks for the asterisk; if there is no asterisk it calls a second service macro that performs a regular complex multiplication, otherwise it calls a third service macro that executes the conjugate multiplication.

```

328 \def\MultVect#1by{\@ifstar{\@ConjMultVect#1by}{\@MultVect#1by}}%
329 %
330 \def\@MultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
331 \GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@
332 \@tempdimb\tu@Y\p@
333 \@tempdimc=\td@X\@tempdima\advance\@tempdimc-\td@Y\@tempdimb
334 \Numero\t@X\@tempdimc
335 \@tempdimc=\td@Y\@tempdima\advance\@tempdimc\td@X\@tempdimb
336 \Numero\t@Y\@tempdimc
337 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}%
338 %
339 \def\@ConjMultVect#1by#2to#3{\GetCoord(#1)\tu@X\tu@Y
340 \GetCoord(#2)\td@X\td@Y \@tempdima\tu@X\p@ \@tempdimb\tu@Y\p@
341 \@tempdimc=\td@X\@tempdima\advance\@tempdimc+\td@Y\@tempdimb
342 \Numero\t@X\@tempdimc
343 \@tempdimc=\td@X\@tempdimb\advance\@tempdimc-\td@Y\@tempdima
344 \Numero\t@Y\@tempdimc
345 \MakeVectorFrom\t@X\t@Y to#3\ignorespaces}

```

The division of two complex numbers implies scaling down the dividend by the magnitude of the divisor and by rotating the dividend scaled vector by the opposite direction of the divisor; therefore:

```

346 \def\DivVect#1by#2to#3{\ModAndDirOfVect#2to\@Mod and\@Dir
347 \DivideE\p@ by\@Mod\p@ to\@Mod \ConjVect\@Dir to\@Dir
348 \ScaleVect#1by\@Mod to\@tempa
349 \MultVect\@tempa by\@Dir to#3\ignorespaces}%

```

## 3.6 Arcs and curved vectors

We are now in the position of really doing graphic work.

### 3.6.1 Arcs

We start with tracing a circular arc of arbitrary center, arbitrary starting point and arbitrary aperture; The first macro checks the aperture; if this is not zero it actually proceeds with the necessary computations, otherwise it does nothing.

```

350 \def\Arc(#1)(#2)#3{\begingroup
351 \@tdA=#3\p@ \ifdim\@tdA=\z@ \else
352 \@Arc(#1)(#2)%
353 \fi
354 \endgroup\ignorespaces}%

```

The aperture is already memorized in `\@tdA`; the `\@Arc` macro receives the center coordinates in the first argument and the coordinates of the starting point in the second argument.

```

355 \def\@Arc(#1)(#2){%
356 \ifdim\@tdA>\z@
357 \let\Segno+%
358 \else

```

```

359 \@tdA=-\@tdA \let\Segno-%
360 \fi

```

The rotation angle sign is memorized in `\Segno` and `\@tdA` now contains the absolute value of the arc aperture. If the rotation angle is larger than  $360^\circ$  a message is issued that informs the user that the angle will be reduced modulo  $360^\circ$ ; this operation is performed by successive subtractions rather than with modular arithmetics on the assumption that in general one subtraction suffices.

```

361 \Numero\@gradi\@tdA
362 \ifdim\@tdA>360\p@
363 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
364     and gets reduced\MessageBreak%
365     to the range 0--360 taking the sign into consideration}%
366 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
367 \fi

```

Now the radius is determined and the drawing point is moved to the starting point.

```

368 \SubVect#2from#1to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
369 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY

```

From now on it's better to define a new macro that will be used also in the subsequent macros that trace arcs; here we already have the starting point coordinates and the angle to draw the arc, therefore we just call the new macro, stroke the line and exit.

```

370 \@@Arc
371 \pIIf@strokeGraph\ignorespaces}%

```

And the new macro `\@@Arc` starts with moving the drawing point to the first point and does everything needed for tracing the requested arc, except stroking it; I leave the `stroke` command to the completion of the calling macro and nobody forbids to use the `\@@Arc` macro for other purposes.

```

372 \def\@@Arc{%
373 \pIIf@moveto{\@pPunX\unitlength}{\@pPunY\unitlength}%

```

If the aperture is larger than  $180^\circ$  it traces a semicircle in the right direction and correspondingly reduces the overall aperture.

```

374 \ifdim\@tdA>180\p@
375 \advance\@tdA-180\p@
376 \Numero\@gradi\@tdA
377 \SubVect\@pPun from\@Cent to\@V
378 \AddVect\@V and\@Cent to\@sPun
379 \MultVect\@V by0,-1.3333333to\@V \if\Segno-\ScaleVect\@V by-1to\@V\fi
380 \AddVect\@pPun and\@V to\@pcPun
381 \AddVect\@sPun and\@V to\@scPun
382 \GetCoord(\@pcPun)\@pcPunX\@pcPunY
383 \GetCoord(\@scPun)\@scPunX\@scPunY
384 \GetCoord(\@sPun)\@sPunX\@sPunY
385 \pIIf@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
386     {\@scPunX\unitlength}{\@scPunY\unitlength}%
387     {\@sPunX\unitlength}{\@sPunY\unitlength}%
388 \CopyVect\@sPun to\@pPun
389 \fi

```

If the remaining aperture is not zero it continues tracing the rest of the arc. Here we need the extrema of the arc and the coordinates of the control points of the Bézier cubic spline that traces the arc. The control points lay on the perpendicular

to the vectors that join the arc center to the starting and end points respectively. Their distance  $K$  from the adjacent nodes is determined with the formula

$$K = \frac{4}{3} \frac{1 - \cos \theta}{\sin \theta} R$$

where  $\theta$  is half the arc aperture and  $R$  is its radius.

```

390 \ifdim\@tdA>\z@
391   \DirFromAngle\@gradi to\@Dir \if\Segno-\ConjVect\@Dir to\@Dir \fi
392   \SubVect\@Cent from\@pPun to\@V
393   \MultVect\@V by\@Dir to\@V
394   \AddVect\@Cent and\@V to\@sPun
395   \@tdA=.5\@tdA \Numero\@gradi\@tdA
396   \DirFromAngle\@gradi to\@Phimezzi
397   \GetCoord(\@Phimezzi)\@cosphimezzi\@sinphimezzi
398   \@tdB=1.3333333\p@ \@tdB=\@Raggio\@tdB
399   \@tdC=\p@ \advance\@tdC -\@cosphimezzi\p@ \Numero\@tempa\@tdC
400   \@tdB=\@tempa\@tdB
401   \Divide\@tdB by\@sinphimezzi\p@ to\@cZ
402   \ScaleVect\@Phimezzi by\@cZ to\@Phimezzi
403   \ConjVect\@Phimezzi to\@mPhimezzi
404   \if\Segno-%
405     \let\@tempa\@Phimezzi
406     \let\@Phimezzi\@mPhimezzi
407     \let\@mPhimezzi\@tempa
408   \fi
409   \SubVect\@sPun from\@pPun to\@V
410   \DirOfVect\@V to\@V
411   \MultVect\@Phimezzi by\@V to\@Phimezzi
412   \AddVect\@sPun and\@Phimezzi to\@scPun
413   \ScaleVect\@V by-1to\@V
414   \MultVect\@mPhimezzi by\@V to\@mPhimezzi
415   \AddVect\@pPun and\@mPhimezzi to\@pcPun
416   \GetCoord(\@pcPun)\@pcPunX\@pcPunY
417   \GetCoord(\@scPun)\@scPunX\@scPunY
418   \GetCoord(\@sPun)\@sPunX\@sPunY
419   \pIIE@curveto{\@pcPunX\unitlength}{\@pcPunY\unitlength}%
420                 {\@scPunX\unitlength}{\@scPunY\unitlength}%
421                 {\@sPunX\unitlength}{\@sPunY\unitlength}%
422 \fi}

```

### 3.6.2 Arc vectors

We exploit much of the above definitions for the `\Arc` macro for drawing circular arcs with an arrow at one or both ends; the first macro `\VerctorArc` draws an arrow at the ending point of the arc; the second macro `\VectorARC` draws arrows at both ends; the arrows have the same shape as those for vectors; actually they are drawn by putting a vector of zero length at the proper arc end(s), therefore they are styled as traditional or PostScript arrows according to the option of the `pict2e` package.

But the specific drawing done here shortens the arc so as not to overlap on the arrow(s); the only or both arrows are also lightly tilted in order to avoid the impression of a corner where the arc enters the arrow tip.

All these operations require a lot of “playing” with vector directions, but even if the operations are numerous, they do not do anything else but: (a) determining the end point and its direction ; (b) determining the arrow length as an angular quantity, i.e. the arc amplitude that must be subtracted from the total arc to be drawn; (c) the direction of the arrow should be corresponding to the tangent to the arc at the point where the arrow tip is attached; (d) tilting the arrow tip by half its angular amplitude; (e) determining the resulting position and direction of the arrow tip so as to draw a zero length vector; (f) possibly repeating the same procedure for the other end of the arc; (g) shortening the total arc angular amplitude by the amount of the arrow tip(s) already set, and (h) then drawing the final circular arc that joins the starting point to the final arrow or one arrow to the other one.

The calling macros are very similar to the `\Arc` macro initial one:

```

423 \def\VectorArc(#1)(#2)#3{\begingroup
424 \@tdA=#3\p@ \ifdim\@tdA=\z@\else
425 \@VArC(#1)(#2)%
426 \fi
427 \endgroup\ignorespaces}%
428 %
429 \def\VectorARC(#1)(#2)#3{\begingroup
430 \@tdA=#3\p@
431 \ifdim\@tdA=\z@\else
432 \@VARC(#1)(#2)%
433 \fi
434 \endgroup\ignorespaces}%

```

The single arrowed arc is defined with the following long macro where all the described operations are performed more or less in the described succession; probably the macro requires a little cleaning, but since it works fine I did not try to optimize it for time or number of tokens. The final part of the macro is almost identical to that of the plain arc; the beginning also is quite similar. The central part is dedicated to the positioning of the arrow tip and to the necessary calculations for determining the tip tilt and the reduction of the total arc length; pay attention that the arrow length, stored in `\@tdE` is a real length, while the radius stored in `\@Raggio` is just a multiple of the `\unitlength`, so that the division (that yields a good angular approximation to the arrow length as seen from the center of the arc) must be done with real lengths. The already defined `\@@Arc` macro actually draws the curved vector stem without stroking it.

```

435 \def\@VArC(#1)(#2){%
436 \ifdim\@tdA>\z@
437 \let\Segno+%
438 \else
439 \@tdA=-\@tdA \let\Segno-%
440 \fi \Numero\@gradi\@tdA
441 \ifdim\@tdA>360\p@
442 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
443 and gets reduced\MessageBreak%
444 to the range 0--360 taking the sign into consideration}%
445 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
446 \fi
447 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
448 \@tdE=\pIle@FAW\@wholewidth \@tdE=\pIle@FAL\@tdE

```

```

449 \DivideE\@tdE by \@Raggio\unitlength to\DeltaGradi
450 \@tdD=\DeltaGradi\p@
451 \@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
452 \@tdD=\ifx\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
453 \DirFromAngle\@tempa to\@Dir
454 \MultVect\@V by\@Dir to\@sPun
455 \edef\@tempA{\ifx\Segno-\m@ne\else\@ne\fi}%
456 \MultVect\@sPun by 0,\@tempA to\@vPun
457 \DirOfVect\@vPun to\@Dir
458 \AddVect\@sPun and #1 to \@sPun
459 \GetCoord(\@sPun)\@tdX\@tdY
460 \@tdD\ifx\Segno--\fi\DeltaGradi\p@
461 \@tdD=.5\@tdD \Numero\DeltaGradi\@tdD
462 \DirFromAngle\DeltaGradi to\@DirD
463 \MultVect\@Dir by*\@DirD to\@Dir
464 \GetCoord(\@Dir)\@xnum\@ynum
465 \put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}%
466 \@tdE =\ifx\Segno--\fi\DeltaGradi\p@
467 \advance\@tdA -\@tdE \Numero\@gradi\@tdA
468 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
469 \@@Arc
470 \pIle@strokeGraph\ignorespaces}%

```

The macro for the arc terminated with arrow tips at both ends is again very similar, except it is necessary to repeat the arrow tip positioning also at the starting point. The \@@Arc macro draws the curved stem.

```

471 \def\@VARC(#1)(#2){%
472 \ifdim\@tdA>\z@
473 \let\Segno+%
474 \else
475 \@tdA=-\@tdA \let\Segno-%
476 \fi \Numero\@gradi\@tdA
477 \ifdim\@tdA>360\p@
478 \PackageWarning{curve2e}{The arc aperture is \@gradi\space degrees
479 and gets reduced\MessageBreak%
480 to the range 0--360 taking the sign into consideration}%
481 \@whiledim\@tdA>360\p@\do{\advance\@tdA-360\p@}%
482 \fi
483 \SubVect#1from#2to\@V \ModOfVect\@V to\@Raggio \CopyVect#2to\@pPun
484 \@tdE=\pIle@FAW\@wholewidth \@tdE=0.8\@tdE
485 \DivideE\@tdE by \@Raggio\unitlength to\DeltaGradi
486 \@tdD=\DeltaGradi\p@ \@tdD=57.29578\@tdD \Numero\DeltaGradi\@tdD
487 \@tdD=\ifx\Segno--\fi\@gradi\p@ \Numero\@tempa\@tdD
488 \DirFromAngle\@tempa to\@Dir
489 \MultVect\@V by\@Dir to\@sPun
490 \edef\@tempA{\ifx\Segno-\m@ne\else\@ne\fi}%
491 \MultVect\@sPun by 0,\@tempA to\@vPun
492 \DirOfVect\@vPun to\@Dir
493 \AddVect\@sPun and #1 to \@sPun
494 \GetCoord(\@sPun)\@tdX\@tdY
495 \@tdD\ifx\Segno--\fi\DeltaGradi\p@
496 \@tdD=.5\@tdD \Numero\@tempB\@tdD
497 \DirFromAngle\@tempB to\@DirD
498 \MultVect\@Dir by*\@DirD to\@Dir

```

```

499 \GetCoord(\@Dir)\@xnum\@ynum
500 \put(\@tdX,\@tdY){\vector(\@xnum,\@ynum){0}}%
501 \@tdE =\DeltaGradi\p@
502 \advance\@tdA -2\@tdE \Numero\@gradi\@tdA
503 \CopyVect#1to\@Cent \GetCoord(\@pPun)\@pPunX\@pPunY
504 \SubVect\@Cent from\@pPun to \@V
505 \edef\@tempa{\ifx\Segno-\else-\fi\@ne}%
506 \MultVect\@V by0,\@tempa to\@vPun
507 \@tdE\ifx\Segno--\fi\DeltaGradi\p@
508 \Numero\@tempB{0.5\@tdE}%
509 \DirFromAngle\@tempB to\@Dir
510 \MultVect\@vPun by\@Dir to\@vPun
511 \DirOfVect\@vPun to\@Dir\GetCoord(\@Dir)\@xnum\@ynum
512 \put(\@pPunX,\@pPunY){\vector(\@xnum,\@ynum){0}}
513 \edef\@tempa{\ifx\Segno--\fi\DeltaGradi}%
514 \DirFromAngle\@tempa to \@Dir
515 \SubVect\@Cent from\@pPun to\@V
516 \MultVect\@V by\@Dir to\@V
517 \AddVect\@Cent and\@V to\@pPun
518 \GetCoord(\@pPun)\@pPunX\@pPunY
519 \@@Arc
520 \pIIE@strokeGraph\ignorespaces}%

```

It must be understood that the curved vectors, the above circular arcs terminated with an arrow tip at one or both ends, have a nice appearance only if the arc radius is not too small, or, said in a different way, if the arrow tip angular width does not exceed a maximum of a dozen degrees (and this is probably already too much); the tip does not get curved as the arc is, therefore there is not a smooth transition from the curved stem and the straight arrow tip if this one is large in comparison to the arc radius.

### 3.7 General curves

Now we define a macro for tracing a general, not necessarily circular arc. This macro resorts to a general triplet of macros with which it is possible to draw almost anything. It traces a single Bézier spline from a first point where the tangent direction is specified to a second point where again it is specified the tangent direction. Actually this is a special (possibly useless) case where the general `\Curve` macro could do the same or a better job. In any case...

```

521 \def\CurveBetween#1and#2WithDirs#3and#4{%
522 \StartCurveAt#1WithDir{#3}\relax
523 \CurveTo#2WithDir{#4}\CurveFinish}%

```

Actually the above macro is a special case of concatenation of the triplet formed by macros `\StartCurve`, `\CurveTo` and `\CurveFinish`; the second of which can be repeated an arbitrary number of times.

The first macro initializes the drawing and the third one strokes it; the real work is done by the second macro. The first macro initializes the drawing but also memorizes the starting direction; the second macro traces the current Bézier arc reaching the destination point with the specified direction, but memorizes this direction as the one with which to start the next arc. The overall curve is then always smooth because the various Bézier arcs join with continuous tangents. If a cusp is desired it is necessary to change the memorized direction at the end of

the arc before the cusp and before the start of the next arc; this is better than stroking the curve before the cusp and then starting another curve, because the curve jointure at the cusp is not stroked with the same command, therefore we get two superimposed curve terminations. We therefore need another small macro `\ChangeDir` to perform this task.

It is necessary to recall that the directions point to the control points, but they do not define the control points themselves; they are just directions, or, even better, they are simply vectors with the desired direction; the macros themselves provide to the normalization and memorization.

The next desirable point would be to design a macro that accepts optional node directions and computes the missing ones according to a suitable strategy. I can think of many such strategies, but none seems to be generally applicable, in the sense that one strategy might give good results, say, with sinusoids and another one, say, with cardioids, but neither one is suitable for both cases.

For the moment we refrain from automatic direction computation, but we design the general macro as if directions were optional.

Here we begin with the first initializing macro that receives in the first argument the starting point and in the second argument the direction of the tangent (not necessarily normalized to a unit vector)

```
524 \def\StartCurveAt#1WithDir#2{%
525 \beginngroup
526 \GetCoord(#1)\@tempa\@tempb
527 \CopyVect\@tempa,\@tempb to\@Pzero
528 \pIIf@moveto{\@tempa\unitlength}{\@tempb\unitlength}%
529 \GetCoord(#2)\@tempa\@tempb
530 \CopyVect\@tempa,\@tempb to\@Dzero
531 \DirOfVect\@Dzero to\@Dzero}
```

And this reinitializes the direction after a cusp

```
532 \def\ChangeDir<#1>{%
533 \GetCoord(#1)\@tempa\@tempb
534 \CopyVect\@tempa,\@tempb to\@Dzero
535 \DirOfVect\@Dzero to\@Dzero
536 \ignorespaces}
```

The next macro is the finishing one; it strokes the whole curve and closes the group that was opened with `\StartCurve`.

```
537 \def\CurveFinish{\pIIf@strokeGraph\endgroup\ignorespaces}%
```

The “real” curve macro comes next; it is supposed to determine the control points for joining the previous point (initial node) with the specified direction to the next point with another specified direction (final node). Since the control points are along the specified directions, it is necessary to determine the distances from the adjacent curve nodes. This must work correctly even if nodes and directions imply an inflection point somewhere along the arc.

The strategy I devised consists in determining each control point as if it were the control point of a circular arc, precisely an arc of an osculating circle, a circle tangent to the curve at that node. The ambiguity of the stated problem may be solved by establishing that the chord of the osculating circle has the same direction as the chord of the arc being drawn, and that the curve chord is divided into two parts each of which should be interpreted as half the chord of the osculating circle; this curve chord division is made proportionally to the projection of the tangent

directions on the chord itself. Excluding degenerate cases that may be dealt with directly, imagine the triangle built with the chord and the two tangents; this triangle is straightforward if there is no inflection point; otherwise it is necessary to change one of the two directions by reflecting it about the chord. This is much simpler to view if a general rotation of the whole construction is made so as to bring the curve chord on the  $x$  axis, because the reflection about the chord amounts to taking the complex conjugate of one of the directions. In facts with a concave curve the “left” direction vector arrow and the “right” direction vector tail lay in the same half plane, while with an inflected curve, they lay in opposite half plains, so that taking the complex conjugate of one of directions re-establishes the correct situation for the triangle we are looking for.

This done the perpendicular from the triangle vertex to the cord divides the chord in two parts (the foot of this perpendicular may lay outside the chord, but this is no problem since we are looking for positive solutions, so that if we get negative numbers we just negate them); these two parts are taken as the half chords of the osculating circles, therefore there is no problem determining the distances  $K_{\text{left}}$  and  $K_{\text{right}}$  from the left and right nodes by using the same formula we used with circular arcs. Well... the same formula means that we have to determine the radius from the half chord and its inclination with the node tangent; all things we can do with the complex number algebra and macros we already have at our disposal. If we look carefully at this computation done for the circular arc we discover that in practice we used the half chord length instead of the radius; so the coding is actually the same, may be just with different variable names.

We therefore start with getting the points and directions and calculating the chord and its direction

```
538 \def\CurveTo#1WithDir#2{%
539 \def\@Puno{#1}\def\@Duno{#2}\DirOfVect\@Duno to\@Duno
540 \DistanceAndDirOfVect\@Puno minus\@Pzero to\@Chord and\@DirChord
```

Then we rotate everything about the starting point so as to bring the chord on the real axis

```
541 \MultVect\@Dzero by*\@DirChord to \@Dpzero
542 \MultVect\@Duno by*\@DirChord to \@Dpuno
543 \GetCoord(\@Dpzero)\@Xpzero\@Ypzero
544 \GetCoord(\@Dpuno)\@Xpuno\@Ypuno
```

The chord needs not be actually rotated because it suffices its length along the real axis; the chord length is memorized in `\@Chord`.

We now examine the various degenerate cases, when either tangent is perpendicular to the chord, or when it is parallel pointing inward or outward, with or without inflection.

We start with the  $90^\circ$  case for the “left” direction separating the cases when the other direction is or is not  $90^\circ$  ...

```
545 \ifdim\@Xpzero\p@=\z@
546 \ifdim\@Xpuno\p@=\z@
547 \@tdA=0.666666\p@
548 \Numero\@Mcpzero{\@Chord\@tdA}%
549 \edef\@Mcpuno{\@Mcpzero}%
550 \else
551 \@tdA=0.666666\p@
552 \Numero\@Mcpzero{\@Chord\@tdA}%
553 \SetCPmodule\@Mcpuno from\@ne\@Chord\@Dpuno%
```



```

554 \fi
... from when the “left” direction is not perpendicular to the chord; it might be
parallel and we must distinguish the cases for the other direction ...
555 \else
556 \ifdim\@Xpuno\p@=\z@
557 \@tdA=0.666666\p@
558 \Numero\@Mcpuno{\@Chord\@tdA}%
559 \SetCPmodule\@Mcpzero from\@one\@Chord\@Dpzero%
560 \else
561 \ifdim\@Ypzero\p@=\z@
562 \@tdA=0.333333\p@
563 \Numero\@Mcpzero{\@Chord\@tdA}%
564 \ifdim\@Ypuno\p@=\z@
565 \edef\@Mcpuno{\@Mcpzero}%
566 \fi
... from when the left direction is oblique and the other direction is either parallel
to the chord ...
567 \else
568 \ifdim\@Ypuno\p@=\z@
569 \@tdA=0.333333\p@
570 \Numero\@Mcpuno{\@Chord\@tdA}%
571 \SetCPmodule\@Mcpzero from\@one\@Chord\@Dpzero
... and, finally, from when both directions are oblique with respect to the chord;
we must see if there is an inflection point; if both direction point to the same half
plane we have to take the complex conjugate of une direction so as to define the
triangle we were speaking about above.
572 \else
573 \@tdA=\@Ypzero\p@ \@tdA=\@Ypuno\@tdA
574 \ifdim\@tdA>\z@
575 \ConjVect\@Dpuno to\@Dwpuno
576 \else
577 \edef\@Dwpuno{\@Dpuno}%
578 \fi
The control sequence \@Dwpuno contains the right direction for forming the trian-
gle; we cam make the weighed subdivision of the chord according to the horizontal
components of the directions; we eventually turn negative values to positive ones
since we are intersted in the magnitudes of the control vectors.
579 \GetCoord(\@Dwpuno)\@Xwpuno\@Ywpuno
580 \@tdA=\@Xpzero\p@ \@tdA=\@Ywpuno\@tdA
581 \@tdB=\@Xwpuno\p@ \@tdB=\@Ypzero\@tdB
582 \DivideE\@tdB by\@tdA to\@Fact
583 \@tdC=\p@ \advance\@tdC-\@Fact\p@
584 \ifdim\@tdC<\z@ \@tdC=-\@tdC\fi
585 \DivideE\p@ by \@Fact\p@ to\@Fact
586 \@tdD=\p@ \advance\@tdD-\@Fact\p@
587 \ifdim\@tdD<\z@ \@tdD=-\@tdD\fi
before dividing by the denominator we have to check the directions, although
oblique to the chord are not parallel to one another; in this case there is no
question of a weighed subdivision of the chord
588 \ifdim\@tdD<0.0001\p@

```

```

589         \def\@factzero{1}%
590         \def\@factuno{1}%
591     \else
592         \DividE\p@ by\@tdC to\@factzero
593         \DividE\p@ by\@tdD to\@factuno
594     \fi

```

We now have the subdivision factors and we call another macro for determining the required magnitudes

```

595         \SetCPmodule\@Mcpzero from\@factzero\@Chord\@Dpzero
596         \SetCPmodule\@Mcpuno from\@factuno\@Chord\@Dwpuno
597     \fi
598 \fi
599 \fi
600 \fi

```

Now we have all data we need and we determine the positions of the control points; we do not work any more on the rotated diagram of the horizontal chord, but we operate on the original points and directions; all we had to compute, after all, were the distances of the control point along the specified directions; remember that the “left” control point is along the positive “left” direction, while the “right” control point precedes the curve node along the “right” direction, so that a vector subtraction must be done.

```

601 \ScaleVect\@Dzero by\@Mcpzero to\@CPzero
602 \AddVect\@Pzero and\@CPzero to\@CPzero
603 \ScaleVect\@Duno by\@Mcpuno to\@CPuno
604 \SubVect\@CPuno from\@Puno to\@CPuno

```

Now we have the four points and we can instruct the internal `pict2e` macros to do the path tracing.

```

605 \GetCoord(\@Puno)\@XPuno\@YPuno
606 \GetCoord(\@CPzero)\@XCPzero\@YCPzero
607 \GetCoord(\@CPuno)\@XCPuno\@YCPuno
608 \pIle@curveto{\@XCPzero\unitlength}{\@YCPzero\unitlength}%
609             {\@XCPuno\unitlength}{\@YCPuno\unitlength}%
610             {\@XPuno\unitlength}{\@YPuno\unitlength}%

```

It does not have to stroke the curve because other Bézier splines might still be added to the path. On the opposite it memorizes the final point as the initial point of the next spline

```

611 \CopyVect\@Puno to\@Pzero
612 \CopyVect\@Duno to\@Dzero
613 \ignorespaces}%

```

The next macro is used to determine the control vectors lengths when we have the chord fraction, the chord length and the direction along which to compute the vector; all the input data (arguments from #2 to #4) may be passed as control sequences so the calling statement needs not use any curly braces.

```

614 \def\SetCPmodule#1from#2#3#4{%
615 \GetCoord(#4)\t@X\t@Y
616 \@tdA=#3\p@
617 \@tdA=#2\@tdA
618 \@tdA=1.333333\@tdA
619 \@tdB=\p@ \advance\@tdB +\t@X\p@
620 \DividE\@tdA by\@tdB to#1\relax

```

```
621 \ignorespaces}%
```

We finally define the overall `\Curve` macro that recursively examines an arbitrary list of nodes and directions; node coordinates are grouped within regular parentheses while direction components are grouped within angle brackets. The first call of the macro initializes the drawing process and checks for the next node and direction; if a second node is missing, it issues a warning message and does not draw anything. The second macro defines the path to the next point and checks for another node; if the next list item is a square bracket delimited argument, it interprets it as a change of direction, while if it is another parenthesis delimited argument it interprets it as a new node-direction specification; if the node and direction list is terminated, it issues the stroking command and exits the recursive process. The `@ChangeDir` macro is just an interface for executing the regular `\ChangeDir` macro, but also for recursing again by recalling `@Curve`.

```
622 \def\Curve(#1)<#2>{%
623   \StartCurveAt#1WithDir{#2}%
624   \@ifnextchar\lp@r\@Curve{%
625     \PackageWarning{curve2e}{%
626       Curve specifications must contain at least two nodes!\Messagebreak
627       Please, control your Curve specifications\MessageBreak}}
628 \def\@Curve(#1)<#2>{%
629   \CurveTo#1WithDir{#2}%
630   \@ifnextchar\lp@r\@Curve{%
631     \@ifnextchar[\@ChangeDir\CurveFinish}}
632 \def\@ChangeDir[#1]{\ChangeDir<#1>\@Curve}
```

As a concluding remark, please notice the the `\Curve` macro is certainly the most comfortable to use, but it is sort of frozen in its possibilities. The user may certainly use the `\StartCurve`, `\CurveTo`, `\ChangeDir`, and `\CurveFinish` for a more versatile set of drawing macros; evidently nobody forbids to exploit the full power of the `\cbezier` original macro for cubic splines.

I believe that the set of new macros can really help the user to draw his/her diagrams with more agility; it will be the accumulated experience to decide if this is true.

## References

- [1] Gäßlein H. and Niepraschk R., *The `pict2e` package*, PDF document attached to the “new” `pict2e` bundle; the bundle may be downloaded from any CTAN archive or one of their mirrors.