

Design documents for new features

by the YACAS team ¹

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This book contains some design documents that are in progress, for new features or features or implementation details that need to be changed and improved.

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Chapter 1

Introduction

This book contains some design documents that are in progress, for new features or features or implementation details that need to be changed and improved.

Chapter 2

Algorithms requiring global pattern matchers

2.1 Introduction

The default YACAS transformation rules work on expressions locally. For instance, one could write a transformation rule to replace $1x$ with x . This facility is not suitable for transformation rules that require a more global scope. For instance, this facility does not easily allow one to transform an expression $abca$ to a^2bc .

This type of transformation is often required, and this chapter discusses this issue. The types of transformations required will be discussed, along with possible implementation schemes.

2.2 Types of transformations

Selections within terms

A term is an expression with only factors, subexpressions multiplied. Its general form is $a_1 \dots a_n$ for n factors.

The first, basic type of expression one would like to be able to match is an expression like $\dots a^n \dots a^m \dots$, which in its most general form discovers that an expression only contains multiplications, and that it contains a factor a^n and a factor a^m . We would like this expression to be changed into $\dots a^{n+m} \dots$.

Another example is simplification using trigonometric identities, where a term containing two trigonometric factors is reduced to two terms with each one trigonometric factor. Identities like

$$\cos u \sin v = \frac{1}{2} (\sin(v - u) + \sin(v + u))$$

are easy to write down in one line. Ideally, YACAS would allow a programmer to write this down in one line and then have a global pattern matcher recognize when this pattern is applicable, and apply the transformation rule.

Other examples include:

$$\frac{a^n}{a^m} = a^{n-m}$$

$$\exp(x) \exp(y) = \exp(x + y)$$

and for a given integer k :

$$\frac{(n+k)!}{n!} = (n+k)(n+k-1) \dots (n+1)$$

These are all patterns that could be found within one term containing only factors.

Selections across terms

In addition to being able to match a pattern within one term, it is sometimes useful to be able to recognize patterns across multiple terms. One example is

$$\exp(x) + 2 \exp\left(\frac{x}{2}\right) + 1$$

In this case, it is possible to write this expression as:

$$\left(\exp\left(\frac{x}{2}\right)\right)^2 + 2 \exp\left(\frac{x}{2}\right) + 1 = 0$$

which in turn can be simplified further, by setting

$$y = \exp\left(\frac{x}{2}\right)$$

yielding

$$(y+1)^2 = 0$$

and thus $y = -1$, $\exp\left(\frac{x}{2}\right) = -1$ after factoring. It then follows that the solution is $x = i \cdot 2\pi$:

```
In> Solve((Exp(x/2)+1)^2==0,x)
Out> Complex(0,2*Pi);
In> f(x):=N(Exp(x)+2*Exp(x/2)+1)
Out> True;
In> f(Complex(0,2*Pi))
Out> 0;
```

2.3 Flexible pattern matching

In addition to using more flexible, global pattern matching facilities to simplify expressions, it is sometimes necessary to use such more global pattern matchers for other algorithms also.

For instance, for trigonometric identities, there is sometimes an exact result if the argument is a rational number times π . One would like to be able to recognize $\frac{\pi}{2}$, $\frac{1}{2}\pi$, $\pi\frac{1}{2}$, $4\pi\frac{1}{8}$, and perhaps even 0.5π as being half Pi.

With integration, there is often an antiderivative for an expression if an argument can be written as $ax + b$ where x is the variable being integrated over, and a and b are constants that do not depend on x .

2.4 Towards a solution

As soon as this facility is available, it is expected to be used frequently, and deep in the system. The consequence is that it should be implemented efficiently.

Furthermore, in keeping with the rest of the design of the language, it should be possible to write only a few lines of code in order to specify patterns to be matched and transformations to be performed.

Steps local transformation rules are suited for

Before matching global patterns, the local transformation rules (which are efficient) can be used to do preliminary cleaning up of expressions. Expressions like $a(b+c)d$ can easily be converted to $abd + acd$, or $\frac{a}{b}$ can be easily converted to $\frac{ac}{b}$.

Patterns within terms

For patterns within terms, it is necessary to gather the relevant information from the term in order to be able to decide if a pattern matches. Suppose we want to recognize an expression of the form described in the beginning of this chapter:

$$\dots a^n \dots a^m \dots$$

One solution is to do it in scripts. One needs to gather all the factors. The first option is to write a custom bit of code for this type of transformation, where all factors are gathered in a list, combining the symbol with its exponent. While traversing the expression, one adds items to this list.

For instance for an expression ab^2cda one would first add $(a,1)$, then $(b,2)$, $(c,1)$ and $(d,1)$. Lastly, for the last factor, a , the system could notice that it was already in the list, and just update this item to $(a,2)$. One would end up with the set $((a,2), (b,2), (c,1), (d,1))$. This could be called an internal representation.

After this process finishes, one can traverse this list, producing the required result a^2b^2cd .

This process can be generalized. The steps involved are:

1. traverse the expression to gather information (building an internal representation).
2. for each item, combine it with the internal database of information already gathered. The internal database for this expression will change. The way it will change depends on the type of simplification needed.
3. The internal database representing the required information from the expression is finished. It is an internal representation, however, so it needs to be converted back to a normal expression.

The above scheme is flexible, in that the specific method of adding a sub-expression to the already existing database can be customized for different types of simplification or pattern matching. transformations could even be done in-place. The

specific algorithm for adding a term to the database then acts as a filter.

The difficult part is to choose the form of the database, as that ultimately defines the types of transformations and simplifications that are possible.

When only transformations within one term are needed, the situation is not too difficult.

$$(a[1][1]*\dots*a[1][n1])/(b[1][1]*\dots*b[1][m1])+\dots+(a[p][1]*\dots*a[p][np])/(b[p][1][1]$$

Eg. one could write an expression as a set of terms, where each term is a rational function, consisting of a numerator and denominator, each with just simple factors. One could then process each term, and maintain a database for each term. Thus one could readily simplify $\frac{a^n}{a^m}$ to a^{n-m} , or $\frac{(n+1)!}{n!}$ to $n+1$. The trigonometric simplification scheme described earlier in this chapter would be more sophisticated, as it would require changing one term to two (as a multiplication within a term is changed into an addition of two terms).

Specifying transformation rules in the simplification scheme

The algorithm described in the simplification section requires that factors be added to the database of information on the term the factor belongs to one at a time.

For instance, the database might already have an entry representing a^n . When a new term a^m is encountered, one would want to combine these two into a^{n+m} . This is easy to do with custom code that can directly access the database internal representation.

At the same time, one would ideally want to specify this transformation using notation similar to

$$a^n a^m = a^{n+m}$$

In stead of writing custom hand-written code for this transformation, one would want a system where the a^n part is matched in the database and the a^m part in the expression being processed. The result, a^{n+m} , would be the result, and one would want the system to remove the a^n from the database and put back a^{n+m} .

Actually, for efficiency reasons, one would want the system to change a^n to a^{n+m} in the database. But this is a very specific example where this is easily possible. For transformation rules like

$$\frac{(n + k_{\text{Integer}})!}{n!} = (n + k) \dots (n + 1)$$

one needs a database for a rational function, and one needs to remove one factor and replace it with many other factors.

In the case of the trigonometric identities, one needs to replace something of the form AB to something of the form $C+D$, which requires actually removing the actual term at hand (and not just one factor) and replace it with two terms.

One option is to have the $a^n a^m = a^{n+m}$ simplification be performed automatically, then convert the entire expression into internal representation with one additional facility in the internal representation that one additional collection is made; one can collect factors or terms that one knows will be relevant for the simplification. when simplifying factorials, one can collect all factorials in a special part of the internal representation so they are all grouped together. The algorithm can then proceed to process only these parts. For instance,

$$\frac{(n+2)!ab}{cn!}$$

could be converted to

$$(n+2)!(n!)^{-1}abc^{-1}$$

So that the left side part can be further simplified, resulting in

$$(n+2)(n+1)abc^{-1}$$

Dealing with rational functions

A problem arises with rational functions. In the algorithm described above for traversing an expression, gathering information, the fact that the numerator and denominator of the term being examined can have a greatest common denominator which does not equal one, has not been taken into account.

This can be a problem when the expression is brought into the form described above, as this normalization step would first convert $\frac{a+b}{c}$ to $\frac{a}{c} + \frac{b}{c}$. This breaks up the expression. If $a+b$ and c had common factors, this is now lost.

Thus the rational expression first needs to be scanned for greatest common divisors before being split up.

The problem of finding greatest common divisors is a little bit more complicated than described here. For instance the following expression

$$\frac{x}{x^2-1} + a + \frac{1}{x^2-1}$$

is actually equivalent to

$$a + \frac{1}{x-1}$$

This can only be discovered if the first and the last term are combined into one, leaving out the middle term. Perhaps the fact that the two denominators have a greatest common divisor not equal to one can be used to find terms that can be combined.

Another option is to not split up rational terms when there is an addition involved in the denominator, as little is expected to be gained from the simplification any way after removing greatest common divisors. In stead, the numerator and denominator could be simplified as sub-expressions in their own simplification step.

Global pattern matching in general

Finding patterns in terms has been described above. The idea presented was to have a generic database format in which to collect information about the expression, and then have a routine traverse the expression, applying a filter to each term, bringing all terms to the database.

This scheme might not be the most suitable solution when matching patterns for use in other algorithms. In such cases one is generally interested in whether an expression can possibly be brought into a specific form, which does not necessarily match the form the simplifier brings the expression in.

Suppose we want to find out if an expression matches the form $ax+b$ where both a and b don't depend on x . Furthermore, the algorithm is likely to need the actual values for a and b .

If the algorithm for simplification described earlier were used, one would end up with a list of terms, each with lists of factors in the intermediate format. The information that is needed would be missing. For example:

$$A+B+Cx+D+Ex+F$$

would have to be written down as

$$(C+E)x+A+B+D+F$$

before one can recognize that $ax+b$ fits, and find the exact forms for a and b .

In a sense the method for arriving at this stage is almost the same: first gather information, and then return the information in the requested form. The problem is that this feature, global pattern matching, requires more flexibility.

The fact that the algorithms and internal representation of the simplification scheme described above are not suited for this task can be seen from the following example; suppose we wanted to match $ax+b$ to the expression

$$\frac{A+x+B}{E+D} + E$$

One would want this expression written out as

$$\frac{1}{E+D}x + \frac{A+B}{E+D} + E$$

so that the constants can readily be identified. However, as discussed in the section on rational terms, for simplification it is not wise to split up such rational terms. It is required for this example, however. The filtering step could be made more sophisticated so that this operation can be dealt with.

A more serious concern is with efficiency. Apart from splitting up the expression in too small chunks (one only needs a and b , and thus would only need to gather these), there is another danger: the conversion to internal representation continues until the full expression is processed.

In practice, while the expression is being processed, if custom code were written, the matcher could decide *while* matching, that the pattern will never fit, and terminate immediately. For example, a and b should be independent of x , so if a factor like $\sin x$ is encountered the matching can stop immediately.

If the scheme described in the section dealing with simplification is to be used for general pattern matching also, another step needs to be added to it; the filter operation should be able to terminate the process when it decides early on that code further upstream will fail any way.

Other tools at our disposal

The procedure for simplification offers some interesting opportunities. Apart from the filter returning early on whether it succeeded or failed, one could already have the filter perform transformations on (parts of) the expression, and return the transformed and untransformed parts.

2.5 Composite factors

One important issue with simplification is normalization. Factors can be functions instead of simple variables. In order to decide that $\sin ab$ equals $\sin ba$ and thus simplify $\sin ab - \sin ba$ to zero, it suffices for the system to bring expressions to normal form.

In this case, if $\sin ba$ were first converted to $\sin ab$, the system could readily collect terms and discover that the terms described above cancel each other out and result in zero.

For this, the system needs to impose an order for factors. For this, **LessThan** can be used. **LessThan** defines an ordering for atoms:

```
In> LessThan(a,b)
Out> True;
In> LessThan(b,a)
```

```

Out> False;
In> LessThan(2,a)
Out> True;
In> LessThan(2,1)
Out> False;
In> LessThan(a,"b")
Out> False;

```

Ordering for composed expressions can be created based on the `LessThan` function.

2.6 Simplification in the face of non-commuting algebras

In non-commuting algebras, the rule

$$AB = BA$$

does not hold. This has consequences for a system that is trying to simplify the expression passed in. When gathering information to be stored in the database, what the operation is in fact doing is moving factors around until they are next to each other and can be combined. For instance,

$$A^2BCA$$

would first be re-ordered to

$$A^2ABC$$

which then readily converts to

$$A^3BC$$

Extending this system to also support non-commuting algebra involves disallowing these swaps. For groups of symbols, one could specify that two symbols do not commute. For the above example, supposing A and B do not commute, the resulting expression could then terminate as:

$$A^2BAC$$

One step further would be to support commutation relations. Suppose that after one established that A and B do not commute, that the following relation holds:

$$AB - BA = C$$

Then the part of the expression BA could be converted to $AB - C$. This could then result in the expression:

$$A^2(AB - C)C$$

Which then can be written as:

$$A^3B - A^2C^2$$

Factoring could then yield:

$$A^2(AB - C^2)$$

A potential risk is with the global transformation rules, which do not necessarily adhere to the rules for non-commuting algebras. A special non-commuting multiplication operator could be defined for this to guarantee that this is never a problem.

Chapter 3

How Yacas Deals With Sets of Solutions

3.1 Introduction

(This is a draft)

Worries:

- need to change all code that uses Solve
- need a lot of changes in documentation
- arguments to solve are a bit more verbose than the previous version: `Solve(eq1,eq2,vars)` versus `Solve(eq1 And eq2,vars)`. Suddenly things like `Solve(leftlist==rightlist,vars)` is not possible any more. This has to be done with extra commands (which is ok?). It can not stay the way it was, because lists now mean something else, a collection of disjunct solutions.

The difference between a problem stated and a solution given is a subtle one. From a mathematical standpoint,

```
In> Integrate(x,0,B)Cos(x)
Out> Sin(B);
```

And thus

```
Integrate(x,0,B)Cos(x) == Sin(B)
```

is a true statement. Furthermore, the left hand side is mathematically equivalent to the right hand side. Working out the integration, to arrive at an expression that doesn't imply integration any more is generally perceived to be a more desirable result, even though the two sides are equivalent mathematically.

This implies that the statement of a set of equations declaring equalities is on a same footing as the resulting equations stating a solution:

$$ax + b = c \Rightarrow x = \frac{c - b}{a}.$$

If the value of x is needed, the right hand side is more desirable.

Viewed in this way, the responsibility of a `Solve` function could be to manipulate a set of equations in such a way that a certain piece of information can be pried from it (in this case the value of $x = x(a, b, c)$).

A next step is to be able to use the result returned by a `Solve` operation.

3.2 Implementation Semantics of Solve in Yacas

Suppose there is a set of variables that has a specific combination of solutions and these solutions need to be filled in in an expression: the `Where` operator can be used for this:

```
In> x^2+y^2 Where x==2 And y==3
Out> 13;
```

`Solve` can return one such solution tuple, or a list of tuples. The list of equations can be passed in to `Solve` in exactly the same way. Thus:

```
In> Solve(eq1,var)
Out> a1==b1;
In> Solve(eq1 And eq2 And eq3,varlist)
Out> {a1==b1 And a2==b2,a1==b3 And a2==b4};
```

These equations can be seen as simple simplification rules, the left hand side showing the old value, and the right hand side showing the new value. Interpreted in that way, **Where** is a little simplifier for expressions, using values found by `Solve`.

Assigning values to the variables values globally can be handled with an expression like

```
solns := Solve(equations,{var1,var2});
{var1,var2} := Transpose({var1,var2} Where solns);
```

Multiple sets of values can be applied:

```
In> x^2+y^2 Where {x==2 And y==2,x==3 And y==3}
Out> {8,18};
```

This assigns the the variables lists of values. These variables can then be inserted into other expressions, where threading will fill in all the solutions, and return all possible answers.

Groups of equations can be combined, with

```
Equations := EquationSet1 AddTo EquationSet2
```

or,

```
Equations := Equations AddTo Solutions;
```

Where `Solutions` could have been returned by `Solve`. This last step makes explicit the fact that equations are on a same footing, mathematically, as solutions to equations, and are just another way of looking at a problem.

The equations returned can go farther in that multiple solutions can be returned: if the value of x is needed and the equation determining the value of x is $x \equiv |a|$, then a set of returned solutions could look like:

```
Solutions := { a>=0 And x==a, a<0 And x== -a }
```

The semantics of this list is:

```
either a >= 0 And x equals a, or
a < 0 And x equals -a
```

When more information is published, for instance the value of a has been determined, the sequence for solving this can look like:

```
In> Solve(a==2 AddTo Solutions,{x})
Out> x==2;
```


The solution $a < 0$ And $x == -a$ can not be satisfied, and thus is removed from the list of solutions.

Introducing new information can then be done with the AddTo operator:

```
In> Solutions2 := (a==2 AddTo Solutions);
Out> { a==2 And a>=0 And x==a, a==2
      And a<0 And x==--a };
```

In the above case both solutions can not be true any more, and thus when passing this list to Solve:

```
In> Solve(Solutions2,{x})
Out> x==2;
```

AddTo combines multiple equations through a tensor-product like scheme:

```
In> {A==2,c==d} AddTo {b==3 And d==2}
Out> {A==2 And b==3 And d==2,c==d
      And b==3 And d==2};
In> {A==2,c==d} AddTo {b==3, d==2}
Out> {A==2 And b==3,A==2 And d==2,c==d
      And b==3,c==d And d==2};
```

A list a, b means that a is a solution, OR b is a solution. AddTo then acts as a AND operation:

```
(a or b) and (c or d) =>
(a or b) Addto (c or d) =>
(a and c) or (a and d) or (b and c) or (b and d)
```

Solve gathers information as a list of identities. The second argument is a hint as to what it needs to solve for. It can be a list of variables, but also “Ode” (to solve ordinary differential equations), “Trig” (to simplify for trigonometric identities), “Exp” to simplify for expressions of the form $\exp(x)$, or “Logic” to simplify expressions containing logic. The “Logic” simplifier also should deal with $a > 2 \wedge a < 0$ which it should be able to reduce to **False**.

Solve also leaves room for an ‘assume’ type mechanism, where the equations evolve to keep track of constraints. When for instance the equation $x = \sin y$ is encountered, this might result in a solution set

```
y == ArcSin(x) And x>=-1 And x <= 1
```

3.3 Use Case Scenarios

To be filled in

Chapter 4

Reflection

Chapter 5

Multi-valued expressions

Chapter 6

Assume facilities

Chapter 7

Defining a new function in the kernel

This section will explain how to add a new kernel-level function to YACAS, and will explain why it is this way.

Chapter 8

A user interface for Yacas

In practice, for power users, YACAS is already a convenient system when accessible from the command line. The command line allows one to enter calculations rapidly when the user already knows the commands that are available, and knows what he wants to do and what is possible. This unfortunately includes all developers working on YACAS, so there is little incentive to create a user interface front end for YACAS.

Nevertheless, this chapter will try to specify a graphical user interface that services users other than power users.

The single big usability issue is currently that one already needs to know the system a bit before one can use it. The user is greeted with an intimidating flashing cursor, waiting for the user to enter a command. The user needs to know which commands are available, and when to use those commands. In short, the user, whether it is a new or experienced user, can be greatly helped with information that is more readily available.

8.1 Use case scenarios

There are two use case scenarios the author can think of:

1. The user wants to have some fun, and enjoys playing around with math.
2. The user has a specific calculation that needs to be performed, and hopes YACAS can do it quickly.

The following sections describe the possible features a graphical front end could offer to facilitate these.

8.2 Yacas for fun

When a user is bored and wants to be entertained, and when entertainment includes having fun with math, the user might start up the (currently hypothetical) graphical user interface and start to explore the possibilities. The first question the user will ask is “What can Yacas do?”, and when an interesting subject is found, perhaps play with it, entering various parameters to a calculation model, generate graphs, and maybe even learn something new along the way.

8.3 Yacas for profit

When the user has a real world problem, a calculation that needs to be performed, one that he knows he can do perhaps by looking it up in a book or writing dozens of pages until he reaches the result, or perhaps verify that a calculation is correct, the goal is much narrower. The user already knows what he wants to do, but might want to know *how* he can do it.

8.4 The current solution

With the command line version of YACAS, the solution for the above mentioned use case scenarios is to read the manual, and perhaps try some examples until the user understands the tools available.

The user can then play around with some commands, and finally set up a file with code that will perform the calculation and run that file.

A graphical user interface offers the opportunity to allow the user to access relevant information more quickly than scanning the hundreds of pages of documentation (with the chance of getting lost).

Of course documentation in combination with examples that show how to do specific types of calculations go a long way when a user needs to find out how to do a specific calculation, or wants to know what is possible. However, a user interface might provide the required information more readily.

8.5 Possible additional ways to offer information

“What is available?” information

When typing in a command, it happens often (even with experienced users) that the user forgot the exact arguments to that function, or the order of the arguments.

One solution might be to pop up a tip box with the possible ways the command can be completed.

When the user doesn't know which command to use in the first place, the system could provide a list of possible commands, perhaps categorized by type of operation. For each command a short blurb could be shown about what the routine does, when it is applicable, and perhaps some examples for the user to try out to get a handle on the command.

Arguably the last option is offered by the manual already.

“How do I ...?” information

For more elaborate use, the user might be better off with example calculations. A well-documented example showing how a calculation is done goes a long way.

The user can then use the example as a template for his own calculation. The user interface could even offer a facility to have a template for a calculation, where the user enters some final parameters for that specific calculation. With such a template, most of the work is already done, and all the user needs to do is change some parameters to reflect the calculation he wants to do.

8.6 Other facilities

In addition to the features described above, where a graphical front end offers information in a more flexible way, there are other facilities a graphical front end could offer.

Repeated calculations

Computers are good at doing repetitive tasks. If some task is performed often, it might be a good idea to extend the “template example with parameters” model to actually allow the user to design a user interface for a specific calculation so a calculation using that model can be entered more quickly. Enter a few parameters, and out come the numbers and graphs.

Facilities for programmers

For developers, a good debugger could be handy. The usual facilities like putting breakpoints, stepping through code, seeing (the values of) local variables, could be handy. The command line version already offers a useful command line interactive debugger, so this feature might not be too important.

A tree view of the source code, allowing a programmer to easily navigate through the code could be useful. As a project (and the code body) becomes larger and larger it becomes harder to find things in the scripts.

Chapter 9

The static code analyzer

Yacas has some tools to assess the quality of the scripts. The code checking tools are never finished, as new bugs are found, and guards against them added.

The idea behind the static code checkers is to check that coding standards are upheld, and to mark code that is dangerous and thus is likely to be buggy.

The following sections each describe one specific type of test. The static code analysis code can be found in `codecheck.rep`.

Interface check

As described in an essay elsewhere, files should be careful with what they expose to the environment. the `def` file mechanism and the `LocalSymbols` routine should be used for this. The `interface` check verifies that no global resources are accidentally

The rules that should be upheld are:

1. global variables should not be accessible to the outside world. They should be made local to the module by using `LocalSymbols`.
2. functions can be global, exposed to the outside world, iff they are declared in the corresponding `def` file. Otherwise, they should be made local to the module with `LocalSymbols`.
3. files should not be loaded with `Load` or `Use`, explicitly. Rather, the module should depend on the system automatically loading the right file through the `def` file mechanism.
4. scripts in the standard library should just contain simple function definitions, transformation rules, and initializations of global variables local to that module.

The `interface` check also assumes the code to consist of simple function definitions. It is meant to be used for the scripts in the standard scripts library. Exposing functionality to the outside world is usually less of a problem in one-off scripts to do specific calculations, for instance.

General rules

The following rules are general guide lines.

1. the static code analyzer should be able to recognize, when possible, if variables or functions are defined but not used.
2. functions or variables that are not declared anywhere should be reported, as it might be a typing error. Yacas evaluation just skips the function call, or lets the variable evaluate to itself.
3. platform-specific code is not allowed, in general, and thus use of `SystemCall` or `PlatformOS` or related variables or functions should be reported.

4. macro-like functions with local variables, which use `Eval`, or `Map` or related functions that re-evaluate an expression, should be flagged if the variables are not made unique through `LocalSymbols`.
5. the tools could discover if a variable is masked by another local variable with the same name.

Transformation rule checks

1. ideally there should be a way to determine that type changes are made. For instance, `0*{a,b,c}` should return a list, `{0,0,0}`, and not zero.
2. rules that are completely masked by other rules, and can never be reached, should be flagged.
3. rules with the same precedence, but which are ambiguous, should be reported, as the order in which they are tried might have impact on the result of application of the transformation rules.
4. some form of type checking might be possible, by declaring input types of various functions, and then examining the surrounding code to detect when incorrect types might be passed in. It seems to happen a lot that functions don't verify their input.

Coverage checks

Next to checking if functions are declared in the associated `def` file, the analyzer could also detect if:

1. there is no test code for the function
2. there is no or incorrect documentation for the function

Chapter 10

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